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A CONTRIBUTION TOWARDS AN ALTERNATIVE
MARXIAN THEORY OF FIXED CAPITAL

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The theoretical specification of the competitive rate of profit in the presence of durable capital equipment is an issue that has rightly occupied a place of importance in recent literature. The issues involved are important in their own right: the very concept of the profit rate is linked to and dependent on the conception of depreciation accounting employed. Since the rate of profit must play a crucial role in any theory of capitalist accumulation choices, the theory of fixed capital and its depreciation is a precondition for an explanation of capitalism's "laws of motion." Equally important, the issues involved in conceiving fixed capital have played a pivotal role in the ongoing debates between competing analytical frameworks. The treatment of fixed capital as a joint product has contributed to the burgeoning "classical" critique of neo-classical capital theory, while simultaneously that same treatment has played a role in the debate over the consistency and usefulness of the labor-value categories of traditional Marxian value theory. For both of these reasons, fixed capital and depreciation present problems that should be of concern to every variety of Marxian theory.

The dominant contemporary approach to these problems is, as suggested, the treatment of fixed capital as a case of joint production, an approach revived from it classical roots by Sraffa, and then elaborated in a considerable body of recent literature. The general virtues usually attributed to such an approach are, broadly, two-fold. First, by conceiving the worth of durable equipment in "flow"terms, rather than as a "stock" subject to decay, the joint-product treatment permits an economic interpretation of depreciation superior to any alternative which conceives depreciation as simply a technical phenomenon proceeding at an exogenously given rate. Indeed, it is this refusal to reduce depreciation to
purely technical determinants that allows the joint product approach to handle the determination of the useful life of fixed capital equipment as an endogenous economic problem, rather than as something pre-given by engineering or technological considerations. Second, the method is argued to be absolutely general and consistent, in that "it will give the "correct" answer in every case, no matter how complex, over the life of a durable instrument of production, may be the pattern of failing productivity or increasing maintenance and repairs."² The "correctness" of the answer is defined by Sraffa "in the sense of just fulfilling the . . . condition of making possible the replacement of the means of production and the payment of a uniform rate of profits."³

The approach has several implications, positive and negative, for Marxian thinking. Many Marxian arguments, including some by Marx, have proceeded by means of the simplifying assumption that depreciation can be treated as a kind of linear decay in value terms. On this premise, given the value of a new machine and its known, technically-defined life span of n years, the depreciation quota in any given year is defined as one nth of its original value. Such an approach treats depreciation as determined independently of all the other factors which enter into the determination of production prices and the profit rate. The joint product treatment makes a positive and compelling contribution by arguing that this sort of linear calculation of depreciation represents a technological essentialism that is both logically problematic and unnecessary. However, at the same time, the joint product treatment is associated with a destructive critique of the use of value categories in Marxian arguments. Steedman has advanced this point most polemically, arguing that value calculations are irrelevant in general, but worse, in the presence of fixed capital, value magnitudes may

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be mutually inconsistent or incoherent (negative values for used but useful machinery, etc.).

Each of these points deserves further consideration, after a brief review of the structure of the standard joint-product treatment.

The Joint Product Treatment of Fixed Capital

Compared to the case of a pure circulating capital economy, the presence of durable equipment is handled in the mathematical sense by the inclusion of additional equations. Consider the simplest case, where there is a single machine-using industry, using a single type of machine, within a broader economy in which all other industries (including the machine-producing industry) produce single products with circulating capital only. In the formal sense, technology must be so specified as to allow the addition of a separate equation for each period ("year") in which the durable machine is used. In each such equation, the machine, evaluated at the relevant price for its age, is entered as an input on the left-hand side, along with the other non-durable inputs and labor. On the right-hand side, output is expressed as the sum of the price of the primary produced output plus that of the jointly produced one-year-older machine. In the last year of its use, when the machine economically wears out, only the primary output appears on the right-hand side.

Each such equation is structurally the same, in that in each case the price of the joint outputs is equated to the sum of the total advances made for all production inputs (machines, non-durable inputs, and wages) plus a profit, calculated at the uniform, general rate of profit, on the magnitude of the advanced capital. Thus, if the machine is used for \( t \) years, then, in comparison to the case of a pure circulating capital industry, there are \( (t - 1) \) additional equations (one for each year beyond
the first in which a used machine enters production), but also \((t - 1)\) additional unknowns (the prices of the used machines of each age that enter and/or leave the various production processes). Depreciation in each year can then be simply calculated as the difference between the price of the machine as input and the price of the one-year-older machine that emerges as a joint output.

Given the wage, therefore, the profit rate (uniform across the different "processes" defined by the age of machinery in use) is determined simultaneously with the prices of all goods, including the accounting prices for the used but still useful machines. In effect, the aggregate magnitude of capital - the total price of circulating inputs and new and used machines - is solved for simultaneously with the rate of profit. As already suggested, the prices derived for used equipment are regarded as the "correct" ones precisely because they permit this uniformity of the profit rate irrespective of the complexity of the pattern of input usage over the life of a machine. Depreciation is what it must be in order to permit the conceptually-imposed uniformity of the rate of profit.

To illustrate, consider the following example of a simple economy in which two primary outputs, corn and machines, are produced. Machines are used only in producing corn, and can be used for two years before it is worn out.

<table>
<thead>
<tr>
<th>New Machines</th>
<th>Old Machines</th>
<th>Labor</th>
<th>Corn</th>
<th>New Machines</th>
<th>Old Machines</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>0</td>
<td>12</td>
<td>24</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>5</td>
<td>14</td>
<td>24</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>5</td>
<td>5</td>
<td>32</td>
<td>48</td>
<td>5</td>
</tr>
</tbody>
</table>

Assume that wages are paid in advance, that the real wage per unit labor
is \((1/2)\) unit of corn, and that the price of corn is taken as numeraire
\((P_c = 1)\). The rate of profit, \(r\), the price of a new (zero years old)
machine, \(P_0\), and the price of a used (one year old) machine, \(P_1\), are then
determined by:

\[
\begin{align*}
(1 + 3)(1 + r) &= 5P_0 \\
(7 + 5P_0 + 6)(1 + r) &= 24 + 5P_1 \\
(8 + 5P_1 + 7)(1 + r) &= 24
\end{align*}
\]

The solution, which uniquely allows the uniformity of the profit rate, is:

\[r^* = 40.3\%, \ P_0^* = 1.122, \ P_1^* = 0.422\]

Of course, as already mentioned, from the joint product perspective, the useful life of the machine - the number of years it will actually be used, as distinct from its "physical" life, however defined - is endogenously determined. Indeed, in the logical sense, the issue of the economic life of the machine is treated as prior to the price solution (and, in fact, prior to the choice between competing technical possibilities). In practice, that determination boils down to the comparison of alternative full-system solutions: one must solve a succession of systems, each of which combines the equations for the rest of the economy with a different life-span for the machine (i.e., assuming a finite maximum for the physical life of the machine, the number of equations within the machine-using industry is varied from one through the maximum possible number). Of these, the solution which yields the maximum uniform rate of profit is argued to be the one which competition will enforce.

The economic explanation for this choice of duration for machinery is analogous to the arguments made from this perspective in
justifying the choice between competing techniques. Picture a machine in use for only one year, and a pattern of prices with a uniform profit rate corresponding to this situation (so that the price of a one year old machine is initially zero). If it now becomes possible to use such a machine for an additional year, capitalists will "cost up" the new possibility, at the existing prices, including the zero price for the used machine. If the use of the machine yields at these prices a profit rate for the second year (a "transitional profit rate") greater than the existing general profit rate, then it is argued that profit seeking capitalists will indeed use the machine for a second year. Mathematically, if this transitional rate of profit exceeds the existing rate, then the new general rate of profit produced as the economy competitively adapts to the new situation will be higher than before, and the price of the used machine will be positive. On the other hand, if the transitional profit rate were less than the existing rate, then the extra year of use is insufficiently profitable and would not be implemented. The non-optimality of the second year in this case is signalled by the nature of the solution obtained when the data for the second year are included in a solution for a uniform profit rate: not merely would that rate be lower than previously, but the price of the used machine would be negative. Negative prices for used machines go hand in hand algebraically with a uniform profit rate lower than that competitively obtained with a different duration of use of fixed capital.\(^5\)

(In the example already given, if it were possible to use the machine for a third year, producing 24 corn by means of 5 two-year
old machines, 9 corn, and 16 labor, it is argued that capitalists
would do so, since the transitional profit rate on this new process
would be 41.2% (24-9-8/9+8). Once profit rate uniformity is restored
by competition, the new solution would be $r^* = 40.5\%, P_0^* = 1.124,$
$P_1^* = 0.430, P_2^* = 0.017$, with a higher rate of profit and positive
prices for all used equipment. But, alternatively, if the third
year's use required 10 corn rather than 9 corn, the transitional rate
on the new process would be 33.3% (24-10-8/10+8). The process would
not be implemented, and its undesirability is indicated in the
solution which incorporates the third year process: $r^*$ falls to
38.9%, and $P_2^* = -0.143$, where the negative price signals the
non-optimality of the chosen duration for the machine).

The solution obtained is elegant and determinate, and the
approach as a whole is compelling in its demonstration that
depreciation cannot be logically treated as a purely technical
phenomenon. Consider the special case of a machine of constant
efficiency over its useful life, i.e., the same quantities of other
inputs and labor combined with the machine yield the same amount of
primary produced output, irrespective of the machine's age. In this
case, if each process if to earn profit at the same rate, then the
implicit accounting prices for used machines cannot be calculated on
the premise of straight-line depreciation. With constant efficiency,
the "capital charge" (depreciation plus profit on the magnitude of
machine capital advanced each year) must be constant, since all other
components of output price are constant as the machine ages. But the
profit portion of this capital charge must fall due to the shrinking
base on which profit at the uniform rate is calculated (each used,
partially depreciated machine represents a smaller magnitude of capital. To maintain a constant capital charge with a shrinking profit portion of that charge means that depreciation must rise as the machine ages. Hence, a uniform rate of profit on machines of constant efficiency demands a non-uniform (rising) quota for depreciation as the machine ages, a result incompatible with any sort of a priori rule for the division of the machine's original value over the number of years in its life. 7

Of course, the joint product treatment is argued to be perfectly general, even when the pattern of other inputs and outputs over the machine's life is so varied as to prevent any advance judgment concerning its efficiency. The conceptual basis for this generality deserves examination from an explicitly Marxian perspective.

Evaluation: The Conceptual Basis of the Standard Joint Product Treatment

The problems which a Marxian approach can isolate within this standard joint product treatment are problems of conceptualization, problems made more difficult to express clearly by virtue of the subtleness of the differences which exist between the approaches in the nature of the questions asked, and thus in the logical structure of the answers given. It is curious that the joint product approach - so admirable in its refusal to reduce the economic dimensions of depreciation and machine line to exogenous technical conditions - is nevertheless predicated on other sorts of conceptual reductions. In particular, there are three distinct but related senses in which the approach is founded on an essentialist basis.

First, fixed capital is intentionally and explicitly reduced to
circulating capital. As Pasinetti puts it, "all the elements of the analytical scheme are reduced to flows." As previously noted, this is in one sense precisely the strength of the approach, since it allows a departure from the marginalist notion of an exogenously given "stock of capital." But the strength is also a weakness, in that no dimension of "fixity" remains in the analysis. The fixity of fixed capital is not simply a matter of the temporal durability of a physical machine. From a Marxian perspective, the relevant sense of fixity is also in part social - the fixity in terms of ownership of a machine which, once used, is generally exchangeable only with difficulty. The "market" for used equipment is largely hypothetical, but the analytical reduction of fixed to circulating capital, by imposing an equalized rate of profit on all ages of equipment, proceeds "as if" there were no such asymmetry between fixed and circulating capital. Put another way, the problem of the production price of a unit of fixed capital differs from the circulating capital case in part precisely due to the lack of any corresponding actual market price for the typical used machine, but the standard joint product treatment must treat this as of no analytical significance.

Second, with the "process" as the basic unit of analysis, other potentially significant units of analysis cease to have any analytical role to play, since they are effectively reduced to simple aggregations of the essential component processes. In particular, if the equalization of the profit rate is treated as occurring on the level of competing processes using different ages of durable equipment, then "the firm" or "the industry" must earn that rate of profit by definition, since a firm or industry can be nothing other
than an aggregate of these equally profitable processes. The point is
significant, since it concerns the conception of the units over which
the rate of profit is equalized by competition.

Third and closely related, capitalist competition is itself
reduced to a temporal process defined in terms of its outcome: the
uniformity of the rate of profit across every distinguishable
economic unit. Rather than understanding the equalization of the
profit rate as one among several tendencies inherent in capitalist
competition, the joint-product treatment makes of competition no more
and no less than the process which generates this effect. Competition
thus becomes "a process with a purpose," and the price theory
appropriate to a competitive situation is then one which comprehends
competition by capturing its "end-point": a situation in which all
differences of profitability have been eliminated.

From a Marxian perspective, there is both irony and challenge in
the presence of these conceptual reductions: the standard joint
product treatment of fixed capital is rife with essentialisms, but
without some sort of flow treatment, one must either abdicate on the
issues or return to the technological essentialism associated with
handling depreciation as a sort of technically-define "decay."

Is there an alternative, some other means of employing the joint
product principle? I think there is, but a precondition for such an
alternative is a change in the conceptualization of what "the" rate
of profit is. The three conceptual reductions noted above are all
ultimately founded on and made necessary by the conception of the
profit rate as a general, uniformly present profit rate earned by
every productive unit which is competitively relevant in the economy.
Used machine prices and depreciation are what they must be in order to be compatible with profit rate uniformity across processes. The "firm" or "industry" must cease to have analytical significance once technically-defined processes become the units across which price competition functions to equalize returns. And given that mathematical structure of profit rate uniformity, competition must be understood as the process which produces this result.

The Marxian conception of the profit rate as an average rate, encompassing differences among, and even within, individual capitals, is simply different from the Sraffian conception of the profit rate as both uniform and general. That difference has perhaps been insufficiently stressed in the recent literature. As a means to begin discussion of the difference, we can note a point on which everyone would agree: market prices are not the same as production prices. Because of this, "the" rate of profit is not earned by every capitalist "unit" at every point in time. Conceptual differences begin to emerge when it is noted that Marx would argue that "the" rate of profit would not be earned by every unit even if production prices were hypothetically to reign.

A difference exists even in the case of an economy with single-product industries using only circulating capital. When Marx considers the profit rate across different industries or spheres of production, he understands those industries to be composed of different firms which develop unevenly, and which therefore in general have different degrees of technical efficiency due to their quite possibly different technical processes or forms of social organization. Marx's desired solution thus seeks production prices
associated with an average rate of profit - one equalized across industries based on production conditions which are "the average in the total capital employed in [each] particular sphere."\textsuperscript{10} Because of this, it is likely that no particular firm actually earns "the" rate of profit, since any set of prices, including those associated with a profit rate equalized across industries, is consistent with a range of positive and negative surplus profits for the different firms producing each commodity. Production prices so derived can be understood as "centers of gravity" for actual market prices in a "structural" sense, since they are derived by explicitly focussing on the structure of "the existing average conditions of social production."\textsuperscript{11}

Those within the Sraffian tradition would proceed quite differently. The solutions sought for prices and the general profit rate are those characteristic of a "long-period equilibrium," presupposing the choice of an optimal technique and abstracting from the particularities of individual firms' production conditions. "Of the known methods of production, the least cost method, at the going wage, would be the one relevant to the calculation of 'natural prices,'"\textsuperscript{12} so that the profit rate so derived is that which would exist once producers have been forced by competitive pressures to adopt the most profitable and efficient of the existing techniques. The profit rate is thus uniform, and the prices associated with it are understood as "centers of gravity" for market prices in a "temporal" sense, i.e., as those toward which market prices would tend through time, under the pressure of competitive forces.

These differences in approach are worth noting, but they are not all that significant in the case of single-product industries without fixed capital. In that simple case, there are no fundamental disagreements about
how competitive choices would be made. Capital flows between industries, differential access to credit based on profitability, and entry and exit of firms are the aggregate processes which tend to equalize industry profit rates, as individual capitalist firms seek to minimize the cost-price of their output in order both to maximize the rate of profit earned and to expand their market shares by underpricing rivals with higher unit costs.

Nevertheless, presented with the same data (including firm-specific information on physical and labor-time inputs and outputs), the Marxian and Sraffian approaches would solve for different "solutions," because they would use the given information in different ways. The Sraffian, viewing competition as a temporal process which eliminates profitability differences in part by enforcing technical conformity, would seek the optimal or "dominant" techniques for producing each commodity. Marx, in contrast, viewing competition as a process which both continuously eliminates and yet continuously recreates differences between capitalist firms, would view "the" profit rate as a structural rather than an intertemporal abstraction, and would thus employ as his technical data a weighted average of the actual techniques in use.¹³

These differences in conceptualization become more complex and more important when fixed capital is explicitly considered. Shaikh has criticized the standard Sraffian handling of choice of technique by demonstrating that, in the presence of fixed capital, it is entirely possible that the technique which minimizes cost-price leads also to a lower rate of profit than available alternatives.¹⁴ Because capitalist competition would force the adoption of such a technique, the standard argument that competitive technical choices will bring about the highest rate of profit is simply inadequate to capture the pressures imposed by capitalist class relations.
Even beyond these issues, though, the standard joint product treatment of fixed capital imposes a uniform rate of profit across ages of capital equipment. Why? The question is not all that often addressed, perhaps because of a tendency to begin discussion with the case of machines of constant efficiency (for which a uniform profit rate is entirely reasonable) and then move on to more complex cases without any further conceptual justification for the method of calculation. An argument of convenience is possible: a single uniform profit rate is simpler mathematically, since it directly preserves the equality between the number of equations and of unknowns. But such a response is obviously not a conceptual justification.

More significantly, a kind of argument by analogy to the pure circulating capital case is possible: profit rates must tend through time towards uniformity because, where differences in profitability exist, competition will bring about changes through time in the supplies and demands which establish prices for the different commodities. Thus, as capitals seek to earn the highest possible rate of profit, their decisions will alter the prices of the various goods in such a way that higher profit rate situations are eroded while lower profit rate situations are enhanced. Uniformity of profit return in each process used is then argued to be the tendential result. The analogy is not perfect, however, since in the case of pure circulating capital every commodity which has a production price is actually traded in a market. In this simpler case, to each production price, theoretically defined, there corresponds a market price determined in an actual market, so that one can justify theoretically the solution for the former by reference to pressures for changes in demand and supply decisions in these actual markets. Used machinery, on the other hand, while
it may occasionally be bought and sold, is in general very illiquid. Markets in used capital equipment may or may not exist and, realistically, even where such market trades do occur, they tend often to be unusual rather than normal cases (e.g., "buyers' markets," bankruptcy sales, etc.).

The standard joint-product treatment must, explicitly or implicitly, regard such questions as irrelevant. As already noted, Sraffa's reference to "the 'correct' answer" makes clear that the used machine prices derived are 'correct' precisely because they are derived 'as if' a market were present, whether or not such a market actually exists. The accounting procedures employed in determining book values for used equipment 'should' mirror the results that a competitive market would produce, even if no such market exists, because, were there to be such a market, the price it would tend to generate must, under competition, be consistent with a uniform rate of profit on useful capital equipment irrespective of its age.

Here we have the ultimate justification for the conception of the profit rawe as uniform: accounting prices for used equipment must be consistent with a uniform profit rate because, if they are not, then some sort of potential market transaction exists (a price for the used machine profitable both for a buyer to bid and for a seller to accept) which would put pressure on the 'disequilibrium' accounting price to change. In other words, the existence of a potential market transaction, as a relevant foregone alternative, would force a change in accounting methods in the long-run. Even if an actual market for used machines does not initially exist, it would come into being should the book values of used equipment differ persistently from what the market would have generated. Hence, accounting prices calculated on the basis of a uniform rate of profit remove the necessity for any such market to actually come into being -
the market need not be present for its effects to be felt. The conception of competition implicit in this joint product treatment is indeed a broad one: competition tends through time not merely to eliminate inter-industry profit rate differences (via capital flows), and not merely to eliminate inter-firm profit rate differences (via choice of technique), but even to eliminate intra-firm profit rate differences (differences in the rate of profit obtainable with different ages of capital equipment within the same firm or between firms) via the pressures of potential markets which enforce accounting procedures based on profit rate uniformity.

On the simplest level then, in terms of the logic of the solution derived by this method, any machine that would actually be used in the long-run must earn "the" rate of profit. The result is the three types of reductionism noted above. Fixed capital is quite literally reduced to circulating capital, because used machine prices are calculated to be what they would have to be if each machine was actually traded at the end of every production period. The firm and the industry are reduced to simple aggregations of the technical processes of which they are composed. And each of these reductions rests ultimately on a third, the conception of capitalist competition as a kind of entropic process which tends towards uniformity on every level at which profitability can be conceived.

There is power to this treatment, the power of a systematically and rigorously essentialist approach, but the recognition of that power does not mean that no alternative exists. What response can be given by the Marxian approach which conceives "the" rate of profit as an average rate? The issues involved can be illustrated with a question.

Consider an industry which uses a single type of machine, one which declines in efficiency over its economically useful life. For simplicity, let
the declining efficiency be clear and unambiguous (i.e., older machines simply require more of the same types of non-machine inputs per unit output, as in the case illustrated in Table I above). Picture now two capitalist firms, one with a set of relatively new machines, the other with a set of relatively older machines (different age "plants" of the same type, where the older plant continues to be used even though it is simply less efficient in a physical sense). Would not any reasonable capitalist anticipate that even under competitive conditions the older plant would and should earn a lower profit rate than the newer one? To put the same question slightly differently, would not a 'correct' accounting of used machines be one which allows profit rate differentials reflecting the differential efficiency of the plants?

My own answer is yes, and I suspect a poll of "reasonable capitalists" would agree. An older but less efficient plant may be sufficiently profitable to justify its continued use (i.e., the profit rate obtainable on average over the life of an investment in this type of plant is greater with the continued use of the older plant that if that plant were scrapped) without that implying that it must be equally as profitable as newer plants. The conception of competitive profitability expressed here is predicated on a structural relationship between efficiency and profitability, and the "correctness" of the accounting prices for used equipment is defined by their consistency with an efficiency-based structure of positive and negative surplus profits over the life of the machine. The contrast is vivid with the standard joint product treatment in which the "correctness" of depreciation accounting lies precisely in its consistency with a uniform profit rate irrespective of efficiency. Put another way, instead of expressing the effects of efficiency differentials, the standard joint product
treatment must produce prices and depreciation quotas which negate any effect of these efficiency differentials on profit rates.

In contrast, a structural conception of the profit rate, a view of "the" rate of profit as an average rather than uniform rate, would imply a different approach. As in Marx's approach to the pure circulating capital case, we seek a profit rate uniform across the different industries - the rate which, given the production techniques actually in use, equalizes the profitability of the aggregate capitals invested in each industry irrespective of what they produce. Within each industry, however, to the extent that different firms have different structures of inputs and outputs (due to differences in technique, or in organization, or in the age of capital equipment), then the average profit rate is only that - an average of the different rates of profit earned by the differentially efficient firms which compose the industry. Concerning fixed capital then, the differences in relative efficiency which in general result from the use of different age machines would thus imply profit rate differences both among firms (if different firms have differently aged capital equipment) and within the individual firm (if the same firm owns machines of different ages).

An Alternative Approach to the Issues

The preceding section isolated a conceptual difference between "structural" (average profit rate) and "temporal" (uniform profit rate) approaches to the theory of value. The former approach differently poses the question of what solution for prices of production should be, and thus the characteristics and properties of that solution will differ from those produced within the latter. Again, the former approach seeks that rate of profit which is structurally necessary at a point in time (the average rate based on and expressive of the structure of relative efficiencies in use),
rather than a temporally necessary profit rate justifiable as the end product of a hypothetical course of competitive bidding in existing and/or potential markets. It is important therefore to realize that the solutions derived within such a structural approach are not, and are not intended to be, a mimicry of the effects of future market pressures.

There are formidable complexities involved in the pursuit of such a solution. Indeed, the success of the standard joint product treatment in deriving determinate solutions is in part a result of its deliberate abstraction from the complexities which the structural approach outlined above must directly confront. Here I will merely exemplify one approach to deriving the structural rate of profit, one which uses the principle of treating fixed capital as a joint product, but does so in a structurally different fashion so as to avoid the three sorts of reductions pointed out above. Prices and depreciation are solved for in a way which permits only those profit rate differentials justified and made necessary by objectively different levels of efficiency. Of course - and here a debt is owed to the existing joint production literature - there is no way in general to define efficiency independent of prices. Fortunately there is no need to attempt that impossible task since the various profit rates and prices emerge simultaneously.

The notion of a machine with constant physical efficiency provides a kind of benchmark against which differences in efficiency can be specified. In such a special case, the rate of profit would be uniform over the life of the machine, as in the standard joint product treatment. In every other case, profit rate differentials will emerge to mirror the efficiency differentials that exist. Those in turn can be constructed with reference to a transformed pattern of inputs and outputs representing what the industry
would look like using the same aggregate inputs to produce the same aggregate output, but with machines of constant efficiency.

Consider the simple economy represented in Table I, where again the real wage is paid in advance and is equal to \(1/2\) unit of corn per unit labor, and again corn is taken as the unit for relative prices, so that \(P_c = 1\). (As will be seen, this is not simply the normal arbitrary price normalization, since in this example the Marxian production price of corn, in labor-time terms, is also equal to one.)

The actual circulating capital advances (the sum of non-machine constant capital plus wage payments) are, in production price terms, 13 and 15 respectively for the new and used machine processes, while the production price of corn output is in each case 24. Subtracting the former magnitudes from the latter (i.e., taking the difference between the production price of prime output and the production price of the circulating capital advanced) yields 11 and 9 respectively. These numbers represent what can be called the "gross surplus" \(S_t\) for the processes using machines of \(t\) years of age. \(S_t\) expresses, in price of production terms, the excess of primary output over and above the circulating capital payments necessary to produce that output, and therefore represents as a residual the sum of total profits plus depreciation of equipment for the process in question. 17 As such, \(S_t\) provides a measure of the efficiency of the process using machines of age \(t\); since the purpose of fixed capital equipment is to enable a more efficient transformation of circulating capital inputs into final output, then a larger excess of the latter over the former indicates a more efficient production process. Here, due to the simplicity of the example, each \(S_t\) is a single number. In general, though, \(S_t\) would be a complex expression involving multiple types of commodities.
and therefore many prices of production.

Hypothetically, if the machines in use were of constant efficiency, then irrespective of its age each machine would process the same amounts of circulating capital inputs into the same quantity of final output. In the example, that would mean that each process would use its 5 machines to turn 14 units of circulating capital into 24 units of output. On that basis, the "average gross surplus" per process ($\overline{S}_t$) would be 10, where $\overline{S}_t$ states the gross surplus which would be produced by the machines of age $t$ in use if all of the machines used in the industry as a whole were of constant physical efficiency.\textsuperscript{18} Table II summarizes the information so far derived.

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|}
\hline
& circulating capital advanced & $S_t$ & $\overline{S}_t$ & $\Delta S_t$ \\
\hline
new machine process ($t=0$) & 13 & 11 & 10 & +1 \\
\hline
used machine process ($t=1$) & 15 & 9 & 10 & -1 \\
\hline
total & 28 & 20 & 20 & 0 \\
\hline
\end{tabular}
\caption{Table II}
\end{table}

As already argued, a machine of constant efficiency over its life should yield profit at a uniform rate, there being no efficiency differentials to cause positive or negative surplus profits. In the example, however, since efficiency levels differ ($S_t \gtrless \overline{S}_t$), profit rates deviating from the average are to be expected. One way to specify a precise pattern of deviation based on the pattern of differential efficiency is given by the following expression:

$$\frac{r_t - r}{r} = \frac{S_t - \overline{S}_t}{K_t} = \frac{\Delta S_t}{K_t}$$

where $r$ is the average (industry- and economy-wide) profit rate, $r_t$ the
particular profit rate earned on machines of \( t \) years of age, and \( K_t \) the magnitude of (circulating and fixed) capital advanced, in price of production terms, in the process using machines of \( t \) years of age. This relationship states that \( r_t \) will deviate from \( r \) by a percentage (positive or negative) which equals the "extra gross surplus" (\( \Delta S_t \)) produced (positive or negative) per unit of capital advanced. The terms on the right side of the equation are in production price terms, while \( r \) and \( r_t \) are pure numbers. If the industry actually is using constant efficiency machines, then each fraction above is zero, meaning that the average rate of profit is uniformly realized. If not, then \( r_t \) will deviate from \( r \) on the basis of the differential efficiency of its production process.

Cross-multiplying and collecting terms yields

\[
r_t K_t = r K_t + r(\Delta S_t)
\]

This states that the actual profit realized in process \( t \) \( (r_t K_t) \) equals profit at the average rate on its capital advanced \( (r K_t) \) plus an extra or "surplus profit" (positive or negative) equal to profit at the average rate on the extra gross surplus produced. In other words, for a machine which is of above-average efficiency, producing a positive \( \Delta S_t \), in terms of the profit realized it is as if the capitalist had bought the extra gross surplus along with his other capital advances - profit at the average rate is earned on a greater-than-actual amount of capital. Similarly, for a machine of below average efficiency, it is as if a portion of the advanced capital had failed to valorize itself - profit at the average rate is earned on an amount of capital less than that actually advanced.

On this basis it is possible to construct a system which maintains equality between the number of equations and the number of unknowns by calculating profit at the average rate not on the actual capital advances
in the processes which use fixed capital, but on a transformed advance of capital, adjusted to reflect differential efficiency. The price of production equations implied by the relationships above for the machine-using industry have the following form:

\[(K_t + S_t)(1 + r) = \text{(production price of actual output)} + S_t + \text{(production price of used machines produced as a joint product)}\]

Since, by definition, the sum of the \(\Delta S_t\) terms over the total industry using the machine is zero, the \(t\) different equations of this sort representing the \(t\) different machine-using processes must add up to an aggregate equation for the industry which contains only the actual inputs and outputs. The average rate of profit, \(r\), is then the rate actually earned in the industry as a whole, even though each of its component processes may have an actual rate of profit different from this average.\(^{19}\)

To illustrate, in the example, with corn as numeraire, the equations determining \(r\) and the numeraire prices for new machines \((P_0)\) and used machines \((P_1)\) are:

\[(4)(1 + r) = 5P_0\]
\[(13 + 5P_0 + 1)(1 + r) = 24 + 1 + 5P_1\]
\[(15 + 5P_1 - 1)(1 + r) = 24 - 1\]

so that the implied solutions are:

\(r^* = 40.0\%, \ P_0^* = 1.120, \ P_1^* = .486\)

These prices (consistent with the average profit rate for the industry) may be inserted into the equations containing the actual inputs and outputs for the two machine-using processes in order to determine their actual individual profit rates \((r_0^*\) and \(r_1^*\), respectively):

\[(13 + 5(1.120))(1 + r_0) = 25 + 5(.486)\]
\[(15 + 5(.486))(1 + r_1) = 24\]
so that:

\[
\rho_0^* = 42.1\%, \quad \rho_1^* = 37.7\%
\]

Thus, any firm within the industry would earn profits at a rate greater or less than the industry average (i.e., positive or negative surplus profits), depending on whether new or used machines predominate within its capital.

The argument leading up to the solution just derived was expressed in terms of prices of production, Marx's term for the labor-time represented by each commodity in exchange in the presence of the average rate of profit necessitated by capitalist competition. The numerical illustration, however, was carried out in numeraire price terms with corn as numeraire (so that \( P_c = 1 \)). This apparent ambiguity is only apparent, though, because the example is constructed in such a way that production prices and corn numeraire prices are numerically the same. The production conditions specified in Table I have the particular characteristic that the social net product (total physical output, including used machines, minus total physical (non-wage) inputs) is made up of 32 units of corn, numerically equal to the total labor performed (also 32). As I have shown elsewhere, the uniquely Marxian "normalization condition" for production prices as labor-time magnitudes, a condition derived from and based on the concept of value as "socially necessary" labor-time, requires that the total production price of the social net product equal the total amount of new or living labor performed. On this premise, the production price of corn is also equal to one, and the numerical solutions for \( P_0^* \) and \( P_1^* \) above are also the labor-time expressions for the production prices of new and used machines.

In a more general case, the equality between numeraire prices and production prices of course would not hold, but that would in no way affect the determinacy of the solution for either set of magnitudes. Using
the symbol \( p \), with appropriate subscripts, to stand for labor-time production prices allows the specification of the relation between \( P \) and \( P \). If the kth commodity is taken as numeraire, then

\[
P_j = \frac{p_j}{p_k}
\]

where \( P_j \) is any numeraire price and \( p_j \) is any price of production. The solution for production prices can then proceed as outlined above, with numeraire prices following upon specification of the unit of measurement.

Finally, by conceiving the production prices explicitly as labor-time magnitudes, the relation of production prices to values, and of profits to surplus values based on the surplus (unpaid) labor performed, can also be made explicit. Again, as argued in detail elsewhere, the value of a newly-produced commodity - the abstract labor-time socially necessary to reproduce it - is a concept with a meaning which evolves as Marx's analysis of capitalist social relations progresses. Once the concept of an average rate of profit is admitted to the discourse, so that production prices different from values must be calculated, then the constant capital contribution to output value must be understood in price of production terms rather than as a purely technically-defined quantity of physically embodied labor. This occurs because production prices different from values now express the labor-time advance which must be made to procure physical capital, i.e., an advance which is now socially necessary if capitalist production is to proceed at all. In the presence of fixed capital, this means that, to calculate commodity value, both the circulating constant capital consumed (raw and auxiliary materials) and the depreciation quotas for durable equipment must be evaluated in price of production terms. The sum of these two price of production magnitudes (the constant capital component of value) plus the total amount of living
labor performed yields the value of new primary output.

In the numerical example, the value of a machine as a newly produced output \((V_0)\) is given by:

\[
(1) \left( \frac{\rho}{\rho_c} \right) + 6 = 5V_0 \tag{1}
\]

consumed
circulating
capital

\[
\text{total} = \text{total value of new}
\]

living
labor

\[
\text{machine output performed}
\]

\(V_0^* = 1.4\)

Similarly, the value of a unit of corn output \((V_c)\) is determined with reference to the aggregate production conditions of the corn industry:

\[
(15) \left( \frac{\rho}{\rho_c} \right) + \left[ (5\rho_{0} - 5\rho_1) + 5\rho_1 \right] + 26 = 48V_c \tag{15}
\]

consumed
circulating
capital

\[
\text{depreciation} = \text{total value of corn}
\]

living
labor

\[
\text{output performed}
\]

\(V_c^* = 0.971\)

The individual values of the corn produced by each age of machines will of course differ from each other, as always occurs when the same commodity is produced under different production conditions. The social value of corn, however, is determined by the aggregate of the various processes which compose the industry, and is therefore in effect a weighted average of those individual values.

With the values of the two primary outputs calculated in this way, it can easily be shown that the total value of the new commodities produced \((48V_c^* + 5V_0^* = 53.6)\) equals the total production price of those commodities \((48 \rho_c^* + 5 \rho_0^* = 53.6)\). Simultaneously, the total surplus or unpaid labor-time performed in the economy (total labor minus the production price of the aggregate wage bundle: \(32 - \rho_c^*(1/2)32 = 16\)) equals the total profit realized (the average rate of profit times the total capital advanced: \(r(32 \rho_c^* + 5 \rho_0^* + 5 \rho_1^*) = 16.0\)). Lest it be thought that this result is a special case due to the particular production
conditions assumed, it is worth noting that the result is entirely general and is a consequence of the conceptualization of value, production price and the profit rate, rather than the particular conditions to which that conceptualization is here applied.

Summary and Implications

The method of solution proposed here therefore enables one to derive:

(1) the average profit rate (a rate uniformly realized by the aggregate capital of each industry);

(2) the specific and, in general, different individual rates of profit earned by the different machine-using processes within any industry (on the premise that, at any point in time, the relevant "structural" distribution of profit revenues is one which allocates positive and negative surplus profits on the basis of the differential efficiency of the production processes in use);

(3) the labor-time production prices associated with that structure of profit revenues, including the depreciation quotas for each age of durable equipment;

(4) the labor-time values of all newly produced commodities, in specific relation to those production prices;

(5) the numeraire prices proportional to the labor-time production prices, given the choice of numeraire.

The approach is logical, although somewhat arbitrary in the manner of linking profit differential efficiencies. Some such alternative approach is necessary though, if Marxian theory wishes to avoid the sorts of conceptual reductionism on which the standard joint product treatment is based. In the solution derived here, fixed capital is not reduced to circulating capital - capital magnitudes are determined as flows, as in the
more standard approach, but fixed capital is not treated as identical in principle to circulating. Similarly, with profitability differences between processes explicitly allowed for, the firm and the industry exist as units of analysis distinct from the technical processes of which they are composed. And ultimately, both of these characteristics rest upon a conception of capitalist competition as an open-ended process, one which is of course subject to regularities and to particular theoretically explicable pressures, but which is in no sense a simple expression of a single underlying "theme" or "purpose."

From the perspective of the standard joint product approach, it is possible to object that the solution derived here is of little interest since it describes a situation which could not last through time; by the nature of the solution possibilities exist for further competitive changes: through an altered set of technical choices, through a possibly different set of choices on the economic life of machines ("scrapping," truncation), and perhaps even through a sequence of competitive bids which affect the accounting prices of used equipment. Each of these possibilities is real, and yet the objection based upon them is still entirely beside the point, since the solution presented is not intended to describe a lasting situation, or to mimic the pressures towards market equilibrium by abstracting from the present situation to some future state which could last. Competition on a capitalist basis is not a natural process best modelled by analogy to thermodynamic entropy, so that a state of uniformity representing the natural end-point of a process becomes the expression for the process itself. On this point Marx's fundamental insight was that social changes are socially produced, and are explicable only with reference to the social class relations which overdetermine and constitute those changes. Marx
must then view capitalist competition as one process which is inherently
two-sided: the pressures towards uniformity of profitability, technique,
efficiency, growth, etc. are one and the same thing as the pressures
towards non-uniformity, innovation, diversity and change in these same
variables. In this conception therefore it is simply inadequate to attempt to
understand the process as a whole by abstracting from these latter aspects
and using the mathematics of linear price theory to capture the future
outcome of present conditions.

On that basis the solution offered here is, I think, in the Marxian
mold, in that it is designed to illuminate the structure and implications of
what is, at a point in time. By doing so, it may provide an improved basis
for examining the changes that then ensue through time, without reducing
the pressures which lead to such changes to expressions of a
one-dimensional tendency toward uniform profitability. There are important
issues remaining unresolved - choice of technique, scrapping, etc. are
questions this paper has not attempted to deal with. The approach outlined
above can, I think, contribute to the theory required by those questions
(although it remains unclear, to me at least, that these are issues best
handled as a part of the general theory of the profit rate and production
prices).

In any case, the approach outlined permits both a continuation and an
extension of the value-theoretic basis of Marx's analysis of capitalism.
There are in principle no problems involved in extending value categories
to the case of fixed capital, so long as the concept of value is not (as it
is with Steedman, for example) forever frozen into the form appropriate
only to Marx's level of analysis in Volume I of Capital. Both the concept
of value and that of production price (value-form) are jointly required in
order both to specify the linkages between the physical surplus product and
the surplus or unpaid labor extracted and to understand the formation of
an average rate of profit as a process of competitive redistribution among
capitals of that surplus labor extracted in the aggregate through capitalist
class relations. 23 Broadly speaking, then, the presence of fixed capital
opens up a further avenue through which can occur the implicit transfers
of surplus labor-time which give rise to positive and negative surplus
profits within a given industry.

Finally, the Marxian approach as understood here is simply different
from the Sraffian-classical approach concerning what "the" profit rate is,
and therefore concerning the basis on which the algebra of price and value
analysis should be deployed. That there is such a variable as the rate of
profit given prominence in each approach, and that each understands that
profit rate as resting on the existence of a surplus, of course makes
certain common concerns inevitable. Neo-classical marginalism, for example,
is a common enemy of both. However, the fact that "the" profit rate is
differently conceived by the two approaches has been and will continue to
be a source of tension, argument, and ultimately of theoretical
confrontation between alternative frameworks of analysis.
Footnotes


5. The generation of negative prices for used machines is an issue of considerable complexity. Changes in the pre-given distribution parameter (e.g., changes in the real wage) may cause changes in the chosen duration of use for a machine. It is possible also to encounter patterns of input usage such that, for example, given the wage, using a machine for two years yields a negative used machine price and a lesser profit rate than using the machine for one year, but that using the machine for a third year yields positive prices for both vintages of used machines and a higher rate of profit than in either the one or two year cases. See Varri (1980), p. 79.

Similarly, it is entirely possible to have negative depreciation (i.e., a used machine price which exceeds the price of the machine prior to that year's production). Negative depreciation of this sort is not merely a technological phenomenon. Baldone provides an example of a fixed capital system in which varying the distribution of income through its possible range causes changes in the sign of the depreciation allowances, implying "the arbitrariness of formulas of depreciation based on the concept of physical deterioration of the machine" [Baldone (1980), p. 130].

6. The text refers to the joint product treatment as determinate in general, which it is, but special assumptions and/or definitions must be employed in the general case where a particular product is produced through the simultaneous use of two or more machines. Roncaglia points to the nature of the difficulty when he notes that there are "as many different prices associated with a given type of machine at a given point in time as there are feasible 'past histories' for the machine," and that "theoretically the possible 'past histories' are infinite" [Roncaglia (1978), p. 43]. Roncaglia finds a way out of this problem by arbitrarily assuming that the efficiency of each machine (and therefore its price) is independent of its use in the past with other machines of different ages. Varri criticizes this assumptions, and proposes instead "considering the set of machines under consideration as a new, independent 'machine' which is 'composed' of the various individual machines" [Varri (1980), pp. 83-86]. See also Baldone (1980), p. 114, where it is noted that the used 'composite machine' "will have a book value that cannot in general be divided amount its single components." This indivisibility represents a partial indeterminacy, since it must be admitted that any single component of such a composite machine could in principle be separated from the others and sold (at a price which would be completely indeterminate). I might note in passing that the absence of concern demonstrated by these proponents of the joint product approach in this case of partial indeterminacy is perhaps the appropriate response for Marxists when confronted by critics who find it problematic that, in the
case of pure joint products, the values of the joint outputs are indeterminate (because of a similar indivisibility).

7. See Sraffa (1960), pp. 69-70, and also Baldone, as cited in note 5 above.


9. The phrase is intended to invoke Althusser, and especially the critical evaluation he gives to similar views in Althusser (1972), pp. 161-86, and (1976), pp. 94-99.


11. Ibid., p. 641.


13. This is stated or implied by Marx in a multitude of different places, and is made explicit early on in Marx's argument when he notes, regarding commodity value, that it is determined by the labor-time "required to produce an article under the normal conditions of production, and with the average degree of skill and intensity prevalent at the time" (emphasis added) [Marx (1967a), p. 39].

14. Shaikh (1978), pp. 240-46. Comments were offered by Steedman (1980) and others, attempting to contest Shaikh's argument, but, while correct on certain minor points, these comments in no sense rebutted the major contention of Shaikh's original article, as he pointed out in his own reply [Shaikh (1980)].

15. To paraphrase Voltaire, one is forced to conclude that "even if the market did not exist, we would be forced to invent it."


17. The production price of primary output can be decomposed into the sum of the flow of constant capital costs plus variable capital costs plus profit, all in price of production terms. The flow of constant capital costs is, in turn, the sum of circulating constant capital consumed in production plus depreciation of equipment. Thus, subtracting from output production price the sum of circulating capital costs (variable and constant) yields a residual representing profit plus depreciation of equipment.

18. More precisely,  is the  which would result in process if prime output and each type of circulating capital input (including the wage bundle) were, first, summed for the industry as a whole, and then parcelled out to each process in proportion to the percentage of the industry's total number of machines (irrespective of age) which are actually employed in the process (i.e., in proportion to the fraction of the total number of machines which are actually of age )

19. Because of the way  is defined, it contains in a general case negative entries. This is a problem of sorts, since the standard proofs
of the existence and uniqueness of an all-positive solution vector of prices depend upon the Perron-Frobenius theorems applicable only to non-negative matrices. Indeed, it is possible to construct cases in which the method in the text yields economically meaningless solutions (for example, a new machine process with a negative $S$, followed by a used machine process with an enormously high positive $S$, can create the anomalous result of negative profit rates in both). Thus there are limits on the range of conceivable situations which can be dealt with meaningfully. I suspect, without proof, that the range is wide, however, since the anomalies tend to occur in cases which, while not strictly impossible, are nevertheless unlikely and even absurd.


21. See Roberts (1981), chap. III. Note also that the interpretation suggested implies that the rate of profit can be understood as the ratio of total surplus labor-time to the labor-time (price of production) expression for the total capital advanced; on this point, see ibid., pp. 204-05.

22. See Marx (1967a), chap. XII, and also more generally Marx (1967b), p. 761 and p. 833.

References


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